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VECTOR MAGNETIC ANOMALIES DERIVED FROM MEASUREMENTS OF A SINGLE COMPONENT OF THE FIELD†

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Three-component magnetic data are derivable from measurements of one single component of the magnetic field over a plane. The technique involves computation of the double-Fourier-series coefficients of the measured magnetic anomaly, multiplication of the coefficients by a filter operator, and, finally, evaluation of the magnetic components by taking the inverse Fourier transform. The desired filter operator is obtained from a simple relationship between the components of a potential field. The scheme has been tested with excellent results on the fields of a vertical prismatic model.

INTRODUCTION

Magnetic surveys conducted with the purpose of mapping subsurface geologic structures have been largely confined to measurements of a single component of the magnetic field. Most magnetic measurements are made using a total-field instrument, and, since the anomalous field caused by magnetized bodies is normally much smaller than the earth's undisturbed field, the component of the magnetic-field anomaly measured by total-field magnetometers is that which lies in the direction of the main geomagnetic field.

Measurements of a single component of the magnetic-field anomaly over a horizontal plane contain information about the other components. The possibility of deriving three-component data from measurements of one component was first suggested by Vestine and Davids (1945). Hughes and Pondrom (1947) showed that the vertical and horizontal magnetic anomalies can be obtained as surface integrals over the surface of measurement of the total-field anomaly.

The primary purpose of this study is to develop the theory, and a numerical scheme, for transforming a map of the total-field anomaly into three maps, corresponding to each of the magnetic components evaluated along three orthogonal directions. The method is based on the double-Fourier-series expansion of the measured field over an area, using the fast-Fourier-transform (FFT) algorithm for obtaining the coefficients of the expansion. The method has been evaluated using a rectangular vertical-sided prism, a commonly used model in geophysics for simulating the magnetic effects of geologic structures. This has permitted us to make a complete quantitative analysis and to compare the calculated three-component values obtained from the total-field anomaly with the exact three-component values obtained from derived formulas.

Theoretical analyses (Helbig, 1963; Bhattacharyya, 1967) have already pointed out the possibility of determining the magnetization vector, with the help of moments of three-component magnetic data, without any assumption on the shape and size of the disturbing body. The present study indicates the possibility of calculating the direction and magnitude of the magnetization vector from measurements of one single component of the magnetic-field anomaly without

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the necessity of applying Poisson's relation to combined gravity and magnetic data.

THEORY

If ΔV is the anomaly in magnetic potential, at all points of free space, it satisfies the Laplace equation

$$\nabla^2(\Delta V) = 0. \tag{1}$$

The magnitude of the anomalous magnetic-field vector which is superimposed upon the main geomagnetic field, is generally small compared to the earth's undisturbed magnetic field. Therefore, the component of the anomalous field vector measured by total-field magnetometers is assumed to be given by the negative derivative of the anomalous magnetic potential ΔV along the direction of the earth's undisturbed field

$$\Delta \mathbf{T} = -\frac{\partial}{\partial \mathbf{t}}(\Delta V), \tag{2}$$

where $\mathbf{t}$ is the unit vector in the direction of the earth's undisturbed field.

The Cartesian coordinates are chosen such that in the plane of measurements the x-axis points northward, the y-axis eastward, and the $\mathbf{z}$-axis vertically downward. Then the derivative along $\mathbf{t}$ can be expressed as:

$$\frac{\partial}{\partial t} \equiv \alpha_0 \frac{\partial}{\partial x} + \beta_0 \frac{\partial}{\partial y} + \gamma_0 \frac{\partial}{\partial z}, \tag{3}$$

where α_0, β_0, and γ_0 are the direction cosines of the earth's undisturbed field.

Operating on each side of equation (2) with first derivatives, it can be established that

$$\frac{\partial}{\partial x}(\Delta \mathbf{T}) = \frac{\partial}{\partial \mathbf{t}} H_x, \tag{4}$$

$$\frac{\partial}{\partial y}(\Delta \mathbf{T}) = \frac{\partial}{\partial \mathbf{t}} H_y, \tag{5}$$

and

$$\frac{\partial}{\partial z}(\Delta \mathbf{T}) = \frac{\partial}{\partial \mathbf{t}} H_z, \tag{6}$$

where H_x, H_y, and H_z are the components of the anomalous magnetic-field vector given by

$$H_x = -\frac{\partial}{\partial x}(\Delta V), \tag{7}$$

$$H_y = -\frac{\partial}{\partial y}(\Delta V), \tag{8}$$

and

$$H_z = -\frac{\partial}{\partial z}(\Delta V). \tag{9}$$

Our purpose is to use equations (4), (5), and (6) in order to find relationships between the Fourier transform of the total field and the Fourier transforms of the magnetic components. Consequently, the next step will be to represent the total-field anomaly and the magnetic components as a double-Fourier-series expansion.

Since ΔV satisfies Laplace's equation, operating on each side of equation (2) with

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

we have

$$\nabla^2(\Delta \mathbf{T}) = 0. \tag{10}$$

Therefore, $\Delta \mathbf{T}$, under the previous assumption, satisfies Laplace's equation and can be treated by the methods of potential theory. A satisfactory solution to equation (10) in terms of a finite harmonic series expansion is

$$\Delta \mathbf{T}(x, y, z) = \sum_{n=-N/2}^{N/2} \sum_{m=-M/2}^{M/2} W_{T_{mn}} \cdot \exp\left\{2\pi\left(\frac{m^2}{\lambda_x^2} + \frac{n^2}{\lambda_y^2}\right)^{1/2} z\right\} \cdot \exp\left\{2\pi i\left(\frac{mx}{\lambda_x} + \frac{ny}{\lambda_y}\right)\right\}, \tag{11}$$

where λ_x and λ_y are the fundamental wavelengths in the x- and y-directions, and the coefficients $W_{\mathbf{T}_{mn}}$ are the discrete Fourier transforms of $\Delta \mathbf{T}$.

In practice, the magnetic field is of necessity represented as discrete samplings over a finite area. Thus, we digitize the magnetic field on an M by N array, and we stipulate that the field mesh is sufficiently fine and that the data field smoothly approaches zero along the boundary.

The actual field $\Delta \mathbf{T}(x, y, 0)$ is represented in terms of a grid of discrete values:

$$\Delta \mathbf{T}_{jk} \equiv \Delta \mathbf{T}(js, ks)$$

for

$$j = 0, 1, \cdots, M-1,$$

and

$$k = 0, 1, \cdots, N-1,$$

where s is the grid interval.

The fundamental wavelengths λ_x and λ_y are determined by the lateral range of the data and are given by

$$\lambda_x = Ms$$

and

$$\lambda_y = Ns.$$

M is the number of samples along the x-axis, and N is the number of samples along the y-axis.

From the discrete data array $\Delta\mathbf{T}_{jk}$, we obtain the discrete Fourier transform

$$W_{T_{mn}} = \frac{1}{MN} \sum_{j=0}^{M-1} \sum_{k=0}^{N-1} \Delta\mathbf{T}_{jk} \cdot \exp\left\{-2\pi i\left(\frac{jm}{M} + \frac{kn}{N}\right)\right\}. \tag{12}$$

$H_x(x, y, z)$, $H_y(x, y, z)$, and $H_z(x, y, z)$ can also be represented analytically in terms of a finite harmonic series expansion,

$$\begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} = \sum_{n=-N/2}^{N/2} \sum_{m=-M/2}^{M/2} \begin{Bmatrix} W_{x_{mn}} \\ W_{y_{mn}} \\ W_{z_{mn}} \end{Bmatrix} \cdot \exp\left\{2\pi\left(\frac{m^2}{\lambda_x^2} + \frac{n^2}{\lambda_y^2}\right)^{1/2} z\right\} \cdot \exp\left\{2\pi i\left(\frac{mx}{\lambda_x} + \frac{ny}{\lambda_y}\right)\right\}, \tag{13}$$

where the coefficients W_x, W_y, and W_z are the discrete Fourier transforms of the x, y, and z magnetic components, respectively.

Introducing expansions (11) and (13) into equations (4), (5), and (6), performing the derivatives, and comparing term by term the left and right sides of each equation, we get the following relationships:

$$W_{x_{mn}} = i\frac{k_m}{q_{mn}} W_{T_{mn}}, \tag{14}$$

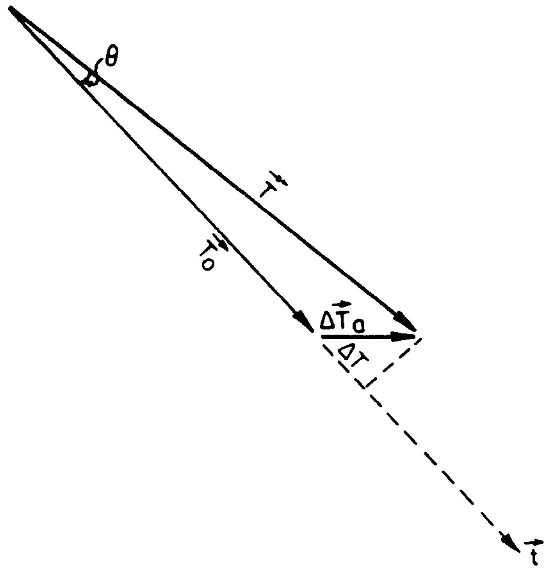

FIG. 1. An anomalous magnetic field vector superimposed upon the main geomagnetic field.

$$W_{y_{mn}} = i\frac{k_n}{q_{mn}} W_{T_{mn}}, \tag{15}$$

and

$$W_{z_{mn}} = \frac{P_{mn}}{q_{mn}} W_{T_{mn}}, \tag{16}$$

where k_m and k_n are the angular frequencies along the x- and y-axis, and are given by

$$k_m = 2\pi m/M$$

and

$$k_n = 2\pi n/N.$$

P_{mn} is given by

$$P_{mn} = 2\pi(m^2/M^2 + n^2/N^2)^{1/2} = (k_m^2 + k_n^2)^{1/2},$$

and q_{mn} is given by

$$q_{mn} = \gamma_0 P_{mn} + i(\alpha_0 k_m + \beta_0 k_n). \tag{17}$$

m and n assume the following values:

$$m = -M/2, \cdots, 0, \cdots, M/2$$

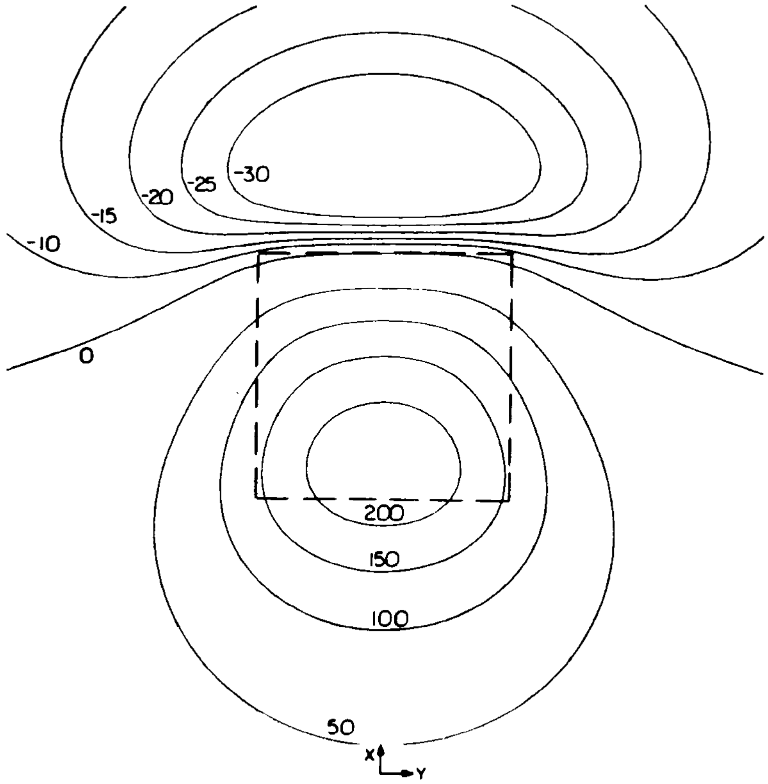

FIG. 2. The total-field anomaly caused by an infinite prism with $a=4$, $b=4$, $h=3$, $D=0$ degrees, $I=60$ degrees, $D_0=0$ degrees, $I_0=60$ degrees. Position of the prism is shown by dashed lines.

and

$$n = -N/2, \cdots, 0, \cdots, N/2.$$

Expressions (14), (15), and (16) give the discrete Fourier transforms $W_{x_{mn}}$, $W_{y_{mn}}$, and $W_{z_{mn}}$ of the magnetic components in terms of the discrete Fourier transform of the total-field anomaly.

It should be noted that those expressions are undefined when m and n are simultaneously equal to zero. Therefore, $W_{x_{00}}$, $W_{y_{00}}$, and $W_{z_{00}}$ have to be calculated by other means as shown in the following paragraphs.

The Fourier transform of function $F(x, y)$, also known as the two-dimensional continuous spectrum, is given by

$$W(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y) \cdot \exp\left\{-2\pi i(ux + vy)\right\} du dv,$$

where u is the frequency (or wavenumber) in the x-direction and v is the frequency (or wavenumber) in the y-direction.

The so-called dc value $W(0, 0)$ is given by

$$W(0, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y) dx dy.$$

If we consider that $F(x, y)$ represents the total field or any magnetic component, then for a finite magnetic source

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y) dx dy = 0.$$

Therefore, for magnetic sources of limited depth extent, the zero-frequency Fourier transforms are given by

$$W_x(0, 0) = W_y(0, 0) = W_z(0, 0) = 0.$$

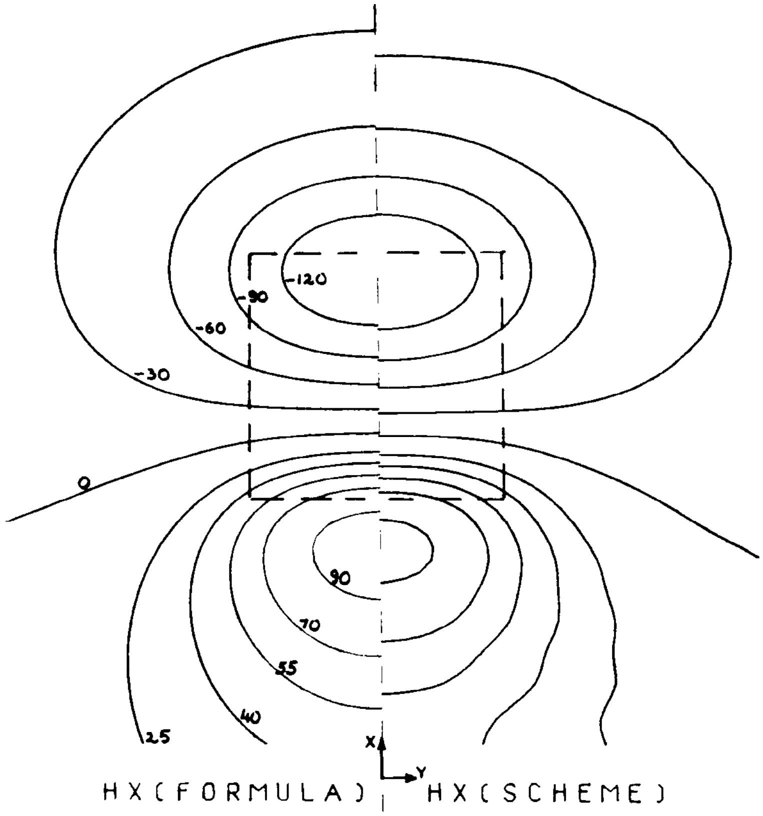

FIG. 3. Magnetic component along the x-axis caused by an infinite prism with $a=4$, $b=4$, $h=3$, $D=0$ degrees, $I=60$ degrees, $D_0=0$ degrees, $I_0=60$ degrees. On the right-hand side of the central line are given the computed values, and on the left-hand, the exact values.

However, whether the sources appear to be depth limited or not will depend very much upon the size of the map. Only if there were no restriction upon the size of the map and the number of samples would the integral above be equal to zero.

Choosing the origin of the Cartesian coordinates to be located at the lower left of the map and assuming that $H_{x_{jk}}$, $H_{y_{jk}}$, and $H_{z_{jk}}$ tend to a constant zero value along the boundary from (13) we get the following expressions:

$$W_{x_{00}} = - \sum_{n=-N/2}^{N/2} \sum_{\substack{m=-M/2 \\ \text{except } m=n=0}}^{M/2} W_{x_{mn}}, \tag{18}$$

$$W_{y_{00}} = - \sum_{n=-N/2}^{N/2} \sum_{\substack{m=-M/2 \\ \text{except } m=n=0}}^{M/2} W_{y_{mn}}, \tag{19}$$

and

$$W_{z_{00}} = - \sum_{n=-N/2}^{N/2} \sum_{\substack{m=-M/2 \\ \text{except } m=n=0}}^{M/2} W_{z_{mn}}. \tag{20}$$

We have set up all the formulation we need. Let us now summarize the steps that lead to the calculation of the three magnetic components, starting from a grid of discrete values of the total-field anomaly.

1) We calculate the discrete Fourier transform $W\mathbf{T}_{mn}$ from the data array $\Delta\mathbf{T}_{jk}$, using equation (12).

2) The complex Fourier coefficients $W_{x_{mn}}$, $W_{y_{mn}}$, and $W_{z_{mn}}$ are obtained from the relationships (14), (15), and (16). $W_{x_{00}}$, $W_{y_{00}}$, and $W_{z_{00}}$ are evaluated from (18), (19), and (20).

3) The final step is to introduce the calculated complex Fourier coefficients $W_{x_{mn}}$, $W_{y_{mn}}$, and $W_{z_{mn}}$ into (13).

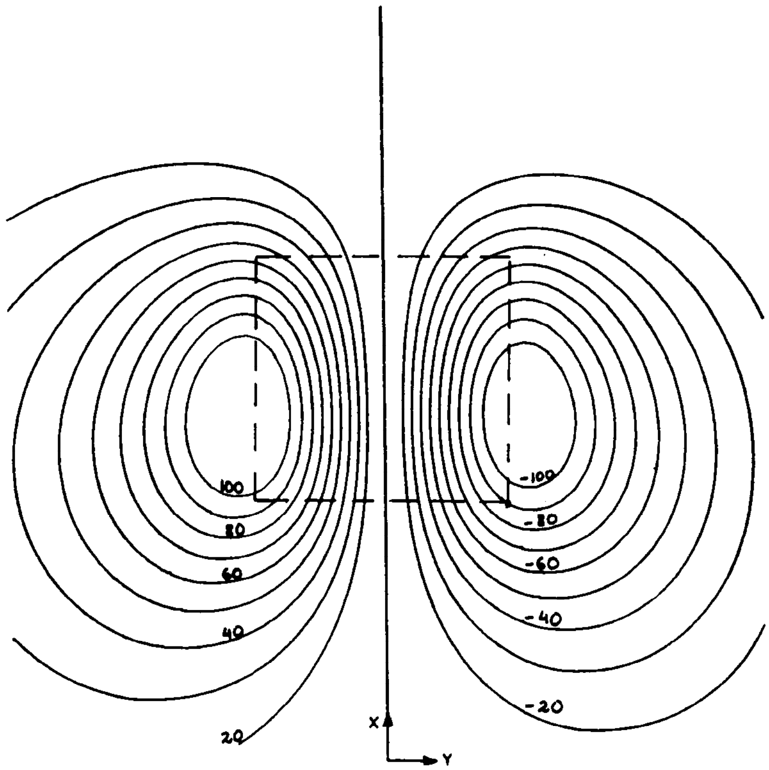

FIG. 4. Magnetic component along the y-axis caused by an infinite prism with $a=4$, $b=4$, $h=3$, $D=0$ degrees, $I=60$ degrees, $D_0=0$ degrees, $I_0=60$ degrees. On the right-hand side of the central line are given the computed values, and on the left-hand side, the exact values.

This process permits the calculation of the components H_x, H_y, and H_z from the total-field values over a plane. Further, it is easily seen from (13) that the vector anomaly can be obtained at any point above or below (but above the source) the measurement plane.

ASSUMPTIONS REGARDING THE TOTAL-FIELD ANOMALY

We have been assuming that the total-field anomaly $\Delta\mathbf{T}$ satisfies Laplace's equation [see equation (10)], and, therefore, it has been treated by the methods of potential theory. In order to obtain Laplace's equation (10), we assumed that [see equation (2)]

$$\Delta\mathbf{T} = -\frac{\partial}{\partial \mathbf{t}}(\Delta V),$$

where $\mathbf{t}$ is a unit vector in the direction of the earth's undisturbed field. However, this equation is not valid unless the total-field anomaly is much smaller than the earth's undisturbed field.

Total-field magnetometers measure the magnitude of

$$\mathbf{T} = \mathbf{T}_0 + \Delta\mathbf{T}_a, \tag{21}$$

where $\mathbf{T}$ is the total-field vector in the vicinity of the magnetized rocks, $\mathbf{T}_0$ is the earth's undisturbed field vector, and $\Delta\mathbf{T}_a$ is the anomalous magnetic-field vector, caused by the magnetized body, and is equal to $-\nabla(\Delta V)$.

Therefore, total-field magnetometers detect the intensity of $\mathbf{T}$ without sensing its direction, and, unless $|\Delta\mathbf{T}_a| \ll |\mathbf{T}_0|$, this direction changes continuously within the vicinity of the magnetized body (Figure 1).

Thus, equation (2) is an approximation and is valid as long as $|\Delta\mathbf{T}_a| \ll |\mathbf{T}_0|$.

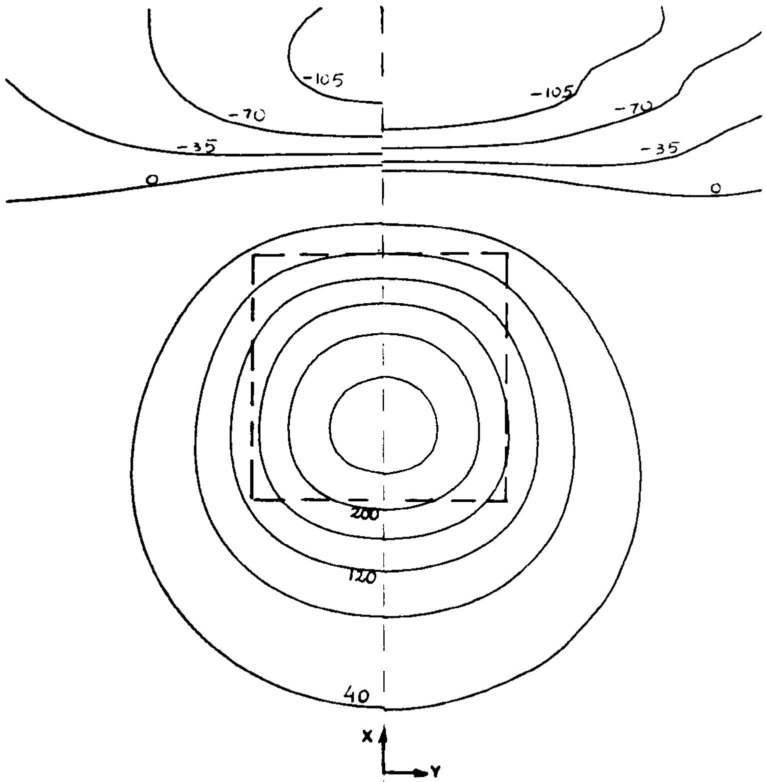

FIG. 5. Vertical magnetic component caused by an infinite prism with $a=4$, $b=4$, $h=3$, $D=0$ degrees, $I=60$ degrees, $D_0=0$ degrees, $I_0=60$ degrees. On the right-hand side of the central line are given the computed values, and on the left-hand side, the exact values.

In practice, this is not a serious limitation on the process of obtaining the anomalous magnetic components. It will be shown, in the following paragraphs, that the error introduced in the computation of the anomalous magnetic components, due to the approximation, may be estimated, and, if necessary, may be corrected by an iteration process, as suggested by Hughes and Pondrom (1947).

Let us derive the exact expression involving the earth's undisturbed field and the total-field anomaly. The starting point is equation (21), which shows the total magnetic vector $\mathbf{T}$ separated into two parts; the anomalous magnetic field vector $\Delta\mathbf{T}_a$ is superimposed upon the main geomagnetic field vector $\mathbf{T}_0$. The components of $\Delta\mathbf{T}_a$ along the x-, y-, and z-axes are H_x, H_y, and H_z.

We may then write

$$\Delta\mathbf{T} = (\mathbf{T} - \mathbf{T}_0) - [H_x^2 + H_y^2 + H_z^2 - (\mathbf{T} - \mathbf{T}_0)^2]/2\mathbf{T}_0. \tag{22}$$

Notice that $(\mathbf{T}-\mathbf{T}_0)$ is the measurement given by total-field magnetometers, after we remove the main geomagnetic field. In the previous sections, the second term of the right-hand side of (22) was neglected and, therefore, it was assumed that

$$\Delta\mathbf{T} = \mathbf{T} - \mathbf{T}_0. \tag{23}$$

This is not an improper approximation in aeromagnetic surveys where anomalies greater than about $0.2\mathbf{T}_0$ are uncommon. As an example, for an anomalous field of the order of 1000 gammas, and assuming $\mathbf{T}_0$ to be of the order of 50,000 gammas, the error in neglecting the second term of equation (22) is at most 10 gammas (1 percent)

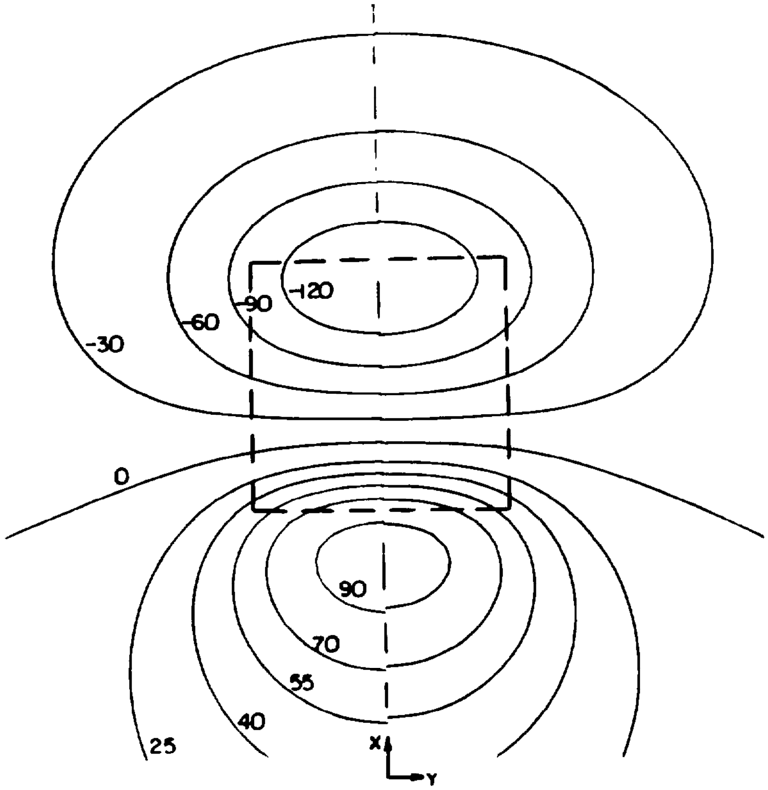

FIG. 6. Magnetic component along the x-axis by an infinite prism with $a=4$, $b=4$, $h=3$, $D=0$ degrees, $I=60$ degrees, $D_0=0$ degrees, $I_0=60$ degrees. On the right-hand side of the central line are given the computed values, and on the left-hand side, the exact values. The original size of the map is 64 by 64.

and is, at most places, much smaller than this amount.

As a matter of fact, we have here a very elementary problem of perturbation theory. In the previous sections we calculated the anomalous magnetic components, H_x, H_y, and H_z, assuming that $\Delta\mathbf{T}$ is given by the first approximation term $(\mathbf{T}-\mathbf{T}_0)$ as shown in (23). Now, with the computed values of H_x, H_y, and H_z, we can estimate the error caused by neglecting the term

$$\epsilon=[H_x^2+H_y^2+H_z^2-(\mathbf{T}-\mathbf{T}_0)^2]/2\mathbf{T}_0. \quad (24)$$

After having evaluated ϵ, we can, if necessary, recalculate $\Delta\mathbf{T}$ using equation (22). The next step is to recalculate the magnetic components, H_x, H_y, and H_z, using now the corrected value of $\Delta\mathbf{T}$. Therefore, through a process of successive iterations, we can improve the calculation of the anomalous magnetic components. In most cases the error ϵ can be neglected, and then it is unnecessary to apply an iteration process.

MODEL RESULTS

The accuracy of the formulation is best checked by comparing the results obtained using the scheme above with the results obtained using the exact values calculated from numerical models. For this purpose we have used the rectangular vertical prism. Three-component data obtained using the above formulation, starting from the finite harmonic series expansion of the given total field, are compared with exact three-component values calculated with exact formulas.

In practice, the total-field anomaly is represented as discrete samplings over a finite area, from which we obtain the corresponding discrete Fourier transform by means of the fast-Fourier-transform computer algorithm (Cooley and Tukey, 1965). The anomalies were calculated on a 32 by 32 array having a unit grid interval. The maps shown in Figures 2 through 5 were contoured using a Calcomp plotter. The contour labels are the magnetic intensity for an arbitrary choice of magnetization of 100. I_0, D_0 are the inclination and declination, respectively, of the earth's field and I, D are the inclination and declination of the magnetization vector. As an example this scheme was applied to the case of an infinite vertical prism, starting from the map of the total magnetic field shown in Figure 2. The horizontal dimensions of the prism are 8 units in the x-direction ($a=4$) and 8 units in the y-direction ($b=4$), and its top is at a depth of 3 units ($h_t=3$). The top is outlined by dashed heavy lines on the contour maps. We have assumed $D_0=D=0$ degrees and $I_0=I=60$ degrees.

Figures 3, 4, and 5 show the magnetic components of the prism calculated by the scheme and also the exact values of the three components. The total-field and exact-component values were derived following Bhattacharyya (1964). On the left side of the central dashed line are the exact values and on the right, the computed values. It is evident that the scheme gives excellent results except, due to the limited map size, along the edges of the plots.

The results can be improved if we increase the size of the map. As an example, Figure 6 shows the calculated magnetic x-component for the same infinite prism, with the total-field data digitized on a 64 by 64 array. This result is considerably better than the result shown in Figure 3. The main reason is that the discontinuities of the magnetic field around the borders of the data grid are smoother for the case of the larger size map.

Note that in these examples we assumed $D_0=D=0$ degrees. This results in symmetry with respect to the x-axis and makes the comparison between computed and exact values less complicated.

GENERALIZATION OF THE METHOD

The formulation and examples that were presented in the first part of this study showed how to derive the anomalous magnetic components, and, therefore, the vector magnetic anomaly, from measurements of the total-field anomaly. The formulation presented there can be easily generalized to include the derivation of the vector magnetic anomaly from measurements of the vertical or any other magnetic component.

For example, if we have the anomalous vertical magnetic field H_z represented as discrete samplings over a finite area, we may obtain the other components. From (14) and (15), when we substitute $W_{t_{mn}}$ by $W_{z_{mn}}$ and make $(\alpha_0, \beta_0, \gamma_0)=(0, 0, 1)$, we obtain

$$W_{x_{mn}} = \frac{ik_m}{p_{mn}} W_{z_{mn}} \tag{25}$$

and

$$W_{y_{mn}} = \frac{ik_n}{P_{mn}} W_{z_{mn}}. \tag{26}$$

$W_{x_{00}}$ and $W_{y_{00}}$ are obtained from equations (18) and (19).

Therefore, the procedure for obtaining the magnetic components H_x and H_y from measurements of the vertical magnetic field is totally analogous to the procedure followed to obtain the magnetic components from measurements of the total field.

CONCLUSIONS

A practical scheme has been presented for the conversion of single-component magnetic data, in particular total-field data, to vector data. This scheme is theoretically straightforward and has been tested numerically for some simple models. This scheme is an extension of the general theory of linear filtering in the frequency domain for the analysis of magnetic anomalies.

The three-component data thus derived have several new applications in magnetic interpretation. As already indicated, the component data obtained from total-field data, may be used to compute moments of the field, which, in turn, are directly related to the vector magnetization of the causative body. The process of reduction to the pole may then be carried out without having to assume a direction of magnetization.

Finally, with three-component data, lines of force of the magnetic anomaly can be plotted in space and are helpful in delineating the causative body (Lourenco, 1972).

We are currently testing these applications with model and field data.

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